Section 3.1 Polynomials

1 term - monomial $\quad 3x, \; y^2, \; -15xyz$

2 terms - binomial $\quad x+y, \; 3x^2-7 \quad 2a^2+7b^2$

3 terms $\rightarrow$ trinomial $\quad \underline{x^2}+\underline{7x}+\underline{10}$

$\underline{a^2}+\underline{b^2}+\underline{c^2}$

Write each polynomial in standard form then give the leading coefficient, the degree, and the number of terms of each?

Decending power of Variable

Leading Coefficent → # out front

Degree → Highest power of the variable

a. $2x - 3x^{\boxed{4}} + 6 - 5x^3$

$-3x^4 - 5x^3 + 2x + 6$

Leading Coefficent $= -3$

Degree $= 4$

# of term $= 4$

b. $x^5 + 2x^6 - 3x^4 - 8x + 4x^3$

$2x^6 + x^5 - 3x^4 + 4x^3 - 8x$

L.C. $= 2$

Degree $= 6$

# of terms: 5

Write each polynomial function in standard form. For each function, find the degree, number of terms, and leading coefficient. $-6x^0$

SEE EXAMPLE 1

$5x^3$

18. $f(x) = -3x^3 + 2x^5 + x + 8x^3 \underline{- 6} + x^4 - 3x^2$

$2x^5 + x^4 + 5x^3 - 3x^2 + x - 6$

D = 5
# of term = 6
L.C. = 2

19. $f(x) = 8x^2 + 10x^7 - 7x^3 - x^4$

$10x^7 - x^4 - 7x^3 + 8x^2$

20. $f(x) = -x^3 + 9x + 12 - x^4 + 5x^2$

$-x^4 - x^3 + 5x^2 + 9x + 12$

How do the sign of the leading coefficient and degree of a polynomial affect the end behavior of the graph of a polynomial function?

↳ Extreme ends

$x \to \infty$ $x \to -\infty$

End Behaviors

$x \to \infty,\ y \to \infty$

$x \to -\infty,\ y \to -\infty$

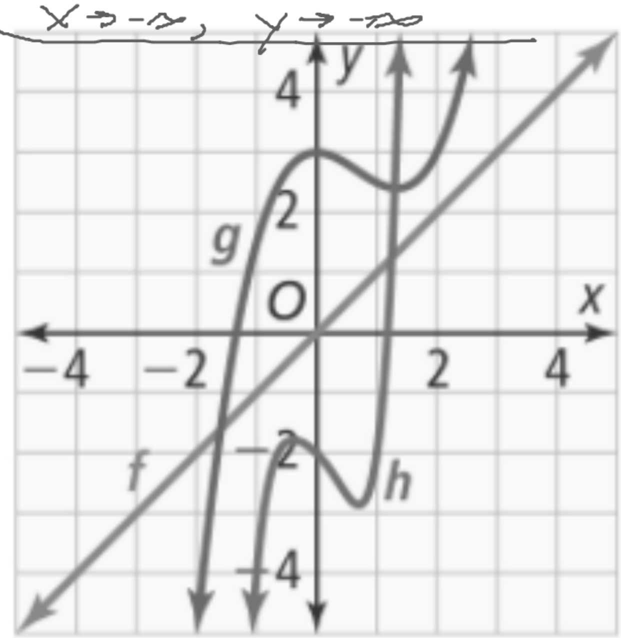
Odd Degree

Positive Leading Coefficient

$f(x) = x$; degree 1

$g(x) = 0.5x^3 - x^2 + 3$; degree 3

$h(x) = 2x^5 - x^2 - x - 2$; degree 5

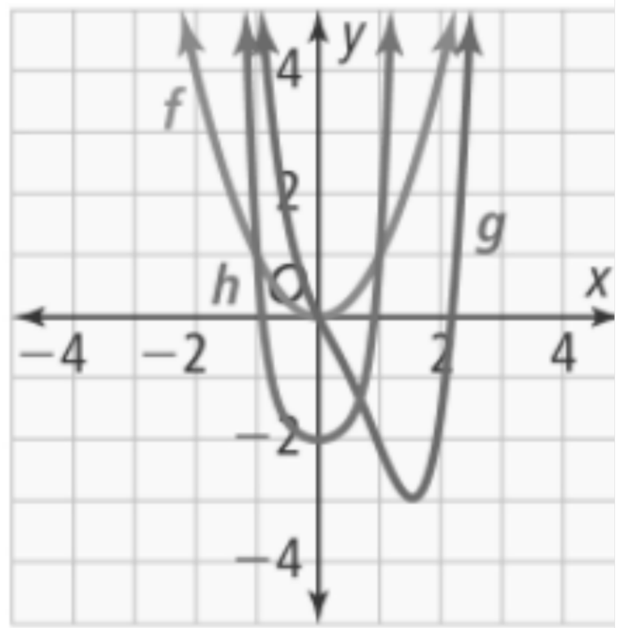


Even Degree
Positive Leading Coefficient

$f(x) = x^2$; degree 2

$g(x) = 0.9x^4 - 2x^3 + x^2 - 3$; degree 4

$h(x) = 2x^6 + x^2 - 2$; degree 6



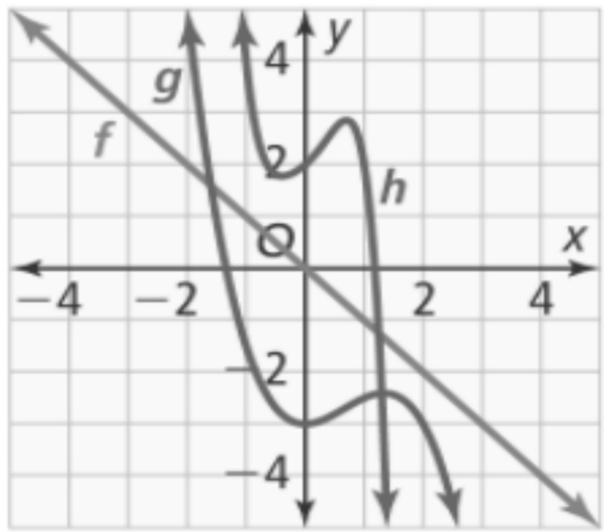
End Behaviors

$x \to \infty,\ y \to \infty$ Right end

$x \to -\infty,\ y \to \infty$ Left end

Odd Degree
Negative Leading Coefficient

$f(x) = -x$; degree 1
$g(x) = -0.5x^3 - x^2 - 3$; degree 3
$h(x) = -2x^5 + x^2 + x + 2$; degree 5



End Behaviors

$x \to \infty$, $y \to -\infty$ Right end

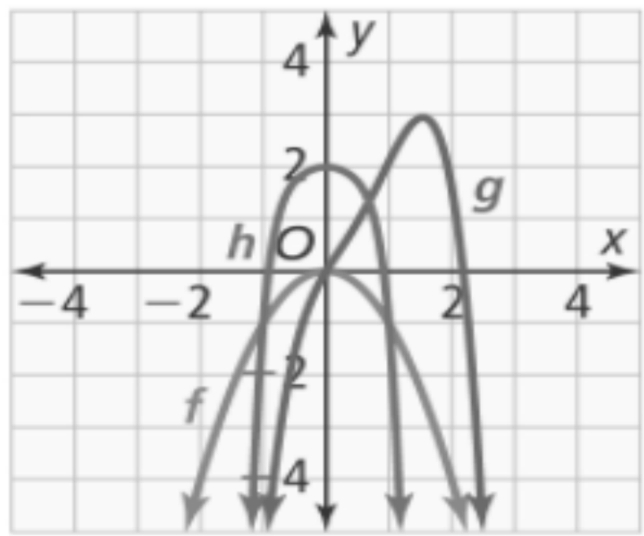
$x \to -\infty$, $y \to \infty$ Left end

Even Degree
Negative Leading Coefficient

$f(x) = -x^2$; degree 2

$g(x) = -0.9x^4 + 2x^3 - x^2 + 2x$; degree 4

$h(x) = -2x^6 - x^2 + 2$; degree 6



End Behaviors

$x \to \infty, y \to -\infty$

$x \to -\infty, y \to -\infty$

2. Use the leading coefficient and degree of the polynomial function to determine the end behavior of each graph.

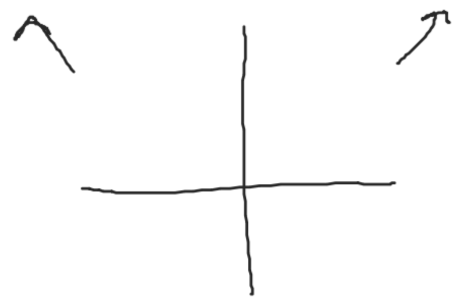
a. $f(x) = 2x^6 - 5x^5 + 6x^4 - x^3 + 4x^2 - x + 1$

Degree → Even

L.C. positive

$x \to \infty, \; y \to \infty$

$x \to -\infty, \; y \to \infty$



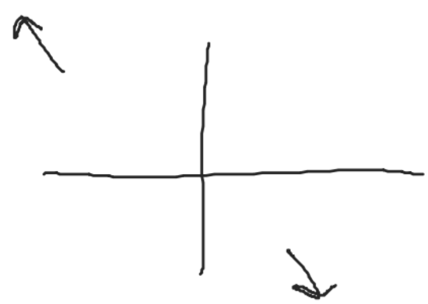
b. $g(x) = -5x^3 + 8x + 4$

Degree → odd

L.C. → Negative

$x \to \infty, \; y \to -\infty$

$x \to -\infty, \; y \to \infty$



Use the leading coefficient and degree of the polynomial function to determine the end behavior of the graph. SEE EXAMPLE 2

21. $f(x) = -x^5 + 2x^4 + 3x^3 + 2x^2 - 8x + 9$

Right $\boxed{x \to \infty, y \to -\infty}$ $\boxed{x \to -\infty, y \to \infty}$ Left

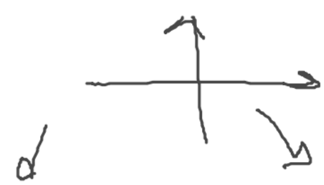
22. $f(x) = 7x^4 - 4x^3 + 7x^2 + 10x - 15$

$x \to \infty$ $y \to \infty$ $x \to -\infty$ $y \to \infty$

23. $f(x) = -x^6 + 7x^5 - x^4 + 2x^3 + 9x^2 - 8x - 2$

$x \to \infty$ $y \to -\infty$

$x \to -\infty$ $y \to -\infty$



Consider the polynomial function $f(x) = -0.5x^4 + 3x^2 + 2$.

Make a table of values and identify intervals where the function is increasing and decreasing.

x	$f(x)$
−3	−11.5
−2	6
−1	4.5
0	2
1	4.5
2	6
3	−11.5

Inc

Dec

Inc

Dec

Dec

$(-1.6, .6)$

$(2.6, \infty)$

Inc

$(-\infty, -1.6)$

$(.6, \boxed{2.6})$

Both ends

$y \to -\infty$

Inc

$(-\infty, -1.7) \cup (0, 1.7)$

Dec

$(-1.7, 0) \cup (1.7, \infty)$

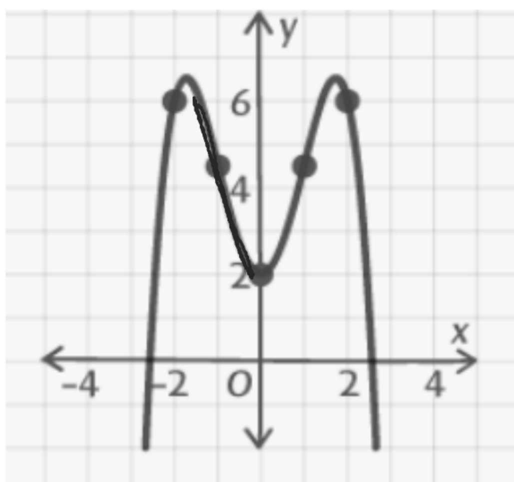
Consider the polynomial function $f(x) = \underline{-0.5x^4 + 3x^2 + 2.}$

B. How can you use the graph to estimate the average rate of change over the interval $[-2,\ 0]$?

Averge Rate of Change

$$\frac{y_2 - y_1}{x_2 - x_1}$$

x	$f(x)$
−3	−11.5
−2	6
−1	4.5
0	2
1	4.5
2	6
3	−11.5



$(-2, 6)$ $(0, 2)$

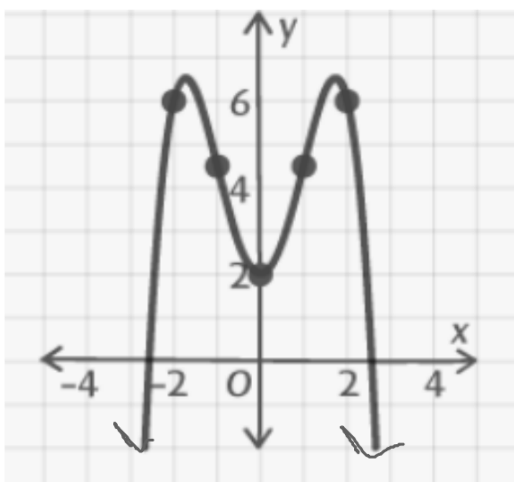
$$ARC = \frac{2-6}{0-(-2)} = \frac{-4}{2}$$

$$= -2$$

Consider the polynomial function $f(x) = -0.5x^4 + 3x^2 + 2$.

Determine the end behavior of the graph

x	$f(x)$
−3	−11.5
−2	6
−1	4.5
0	2
1	4.5
2	6
3	−11.5



$x \to -\infty \quad y \to -\infty$

$x \to \infty \quad y \to -\infty$

3. Consider the polynomial function $f(x) = x^5 + 18x^2 + 10x + 1$.

Make a table of values and identify intervals where the function is increasing and decreasing.

Inc $(-\infty, -1.8) \cup (-.28, \infty)$

Dec $(-1.8, -.28)$

b. Find the average rate of change over the interval [0, 2]

$(0, 1)$ $(2, 125)$

$$\frac{\Delta y}{\Delta x} = \frac{125-1}{2-0} = \frac{124}{2} = 62$$

c. Determine the end behavior of the graph.

$x \to \infty, y \to \infty$

$x \to -\infty, y \to -\infty$

3. Consider the polynomial function $f(x) = x^5 + 18x^2 + 10x + 1$.

Make a table of values and identify intervals where the function is increasing and decreasing.

b. Find the average rate of change over the interval [0, 2]

c. Determine the end behavior of the graph.

3. Consider the polynomial function $f(x) = x^5 + 18x^2 + 10x + 1$.

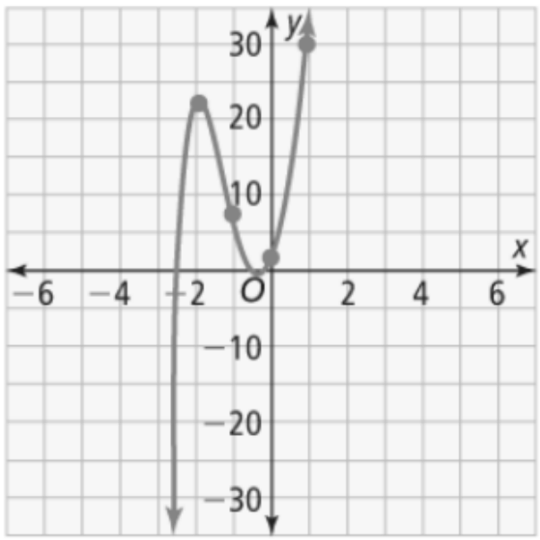
Make a table of values and identify intervals where the function is increasing and decreasing.

b. Find the average rate of change over the interval [0, 2]

c. Determine the end behavior of the graph.

3. Consider the polynomial function $f(x) = x^5 + 18x^2 + 10x + 1$.

x	y
−5	−2724
−4	−775
−3	−110
−2	21
−1	8
0	1
1	30
2	125
3	436
4	1353
5	3626



Use a table of values to estimate the intercepts and turning points of the function. Then graph the function. SEE EXAMPLE 3

24. $f(x) = x^3 + 2x^2 - 5x - 6$

25. $f(x) = x^4 - x^3 - 21x^2 + x + 20$

Points	Intervals
X-intercept	Increasing
Y-Intercept	Decreasing
Local max	$f(x) > 0$
Local min	$f(x) < 0$